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Self-affine roughness influence on the friction coefficient for rubbers onto solid surfaces

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In this paper we investigate the influence of self-affine roughness on the friction coefficient μ_f of a rubber body under incomplete contact onto a solid surface. The roughness is characterized by the rms amplitude *w*, the correlation length ξ, and the roughness exponent *H*. It is shown that with increasing surface roughening at short and/or long length scales (decreasing *H* and/or increasing ratio w/ξ, respectively), the maximum of the friction coefficient μ_f shifts to lower sliding velocities. The latter occurs only for conditions of incomplete contact for small contact length scales λ ($<\xi$). In all cases, the friction coefficient μ_f increases monotonically with decreasing roughness exponent *H* and/or increasing roughness ratio w/ξ and attains its maximum value for sufficiently large contact length scales ($\geqslant\xi$). © *2004 American Institute of Physics.* [DOI: 10.1063/1.1635812]

I. INTRODUCTION

The friction which develops between a rubber body sliding onto a hard solid surface is important from the fundamental and technological point of view. The latter includes the car industry (i.e., tire construction, wiper rubber blades), cosmetic industry, etc.[1–4] The major difference in the frictional properties of rubbers with respect to other solids arise from their low elastic modulus *E*, and the high internal friction that is present over a wide frequency range.[5] The rubber friction is strongly related to its internal friction.[2] At any rate, sliding onto real solid surfaces predominantly occurs on rough surfaces with a significant degree of randomness.[6,7] The latter implies that these surfaces possess roughness over various length scales rather than a single one, which it has to be taken carefully into account in contact related phenomena (i.e., friction and adhesion).[5]

Furthermore, the friction force between a rubber body and a hard rough solid substrate has two contributions which are called hysteric and adhesive.[1] The hysteric arise from the oscillating forces that the surface asperities exert onto the rubber surface leading effectively to cyclic deformations and energy dissipation due to internal frictional damping.[5] As a result the hysteric contribution will have the same temperature dependence as that of an elastic modulus $E(\omega)$.[5] On the other hand, the adhesive component is important for clean and relative smooth surfaces[5] and will not be considered here. In addition, depending on the sliding velocity, the low elastic modulus of rubbers leads to instabilities at high sliding velocities and for relatively smooth surfaces (Schallamach waves[1]). In this case, a compressed rubber surface in front of the contact area undergoes a buckling producing detachment waves from the front-end to the back-end of the contact area. This case will be excluded in the present since it will be limited to low sliding speeds.[5]

Therefore, if rubber body slides with velocity *V* over a sinousoidal rough surface with period *L*, then it will feel fluctuating forces with frequencies $\omega\approx V/L$. Moreover, the contribution of surface roughness to the friction coefficient μ_f at length scales *L* is maximum for relaxation time $\tau\approx L/V$, where the frequency $1/\tau$ is located in the transition regime between rubber (low ω) and glass (high ω) behavior.[5] In addition, if the surface has a wider distribution of length scales *L*, then it will be present a wider distribution of frequency components in the Fourier decomposition of the surface stresses acting on the sliding rubber.[5]

Up to now, it has been shown that for self-affine random rough surfaces, the coefficient of friction μ_f depends significantly on the roughness exponent H $(0\leqslant H\leqslant 1)$, which characterizes the degree of surface irregularity at short length scales.[6,7] Nevertheless, the previous studies were performed using only power law approximations for the self-affine roughness spectrum, which is valid for lateral roughness wavelengths $q\xi>1$ with ξ the in-plane roughness correlation length. This work concentrates on the effect of roughness by inclusion of contributions from roughness wavelengths $q\xi\leqslant 1$ which can be very important for the case of incomplete contact during sliding.

II. THEORY OF FRICTION UNDER CONDITIONS OF INCOMPLETE CONTACT

For a rubber body of Young modulus *E* and Poisson ratio v that slides onto a solid rough surface, if $\lambda=2\pi/q_{\mathrm{con}}$ is of the order of the diameter of the nominal contact area, the coefficient of friction upon sliding with velocity *V* is given by[5]

$$\mu_f=\frac{1}{2}\int_{q_{\mathrm{con}}}^{Q_c}q^3C(q)P(q,q_{\mathrm{con}})dq \times\int_0^{2\pi}\mathrm{Im}\left[\frac{E^*(qV\tau\cos\phi)}{(1-v^2)\sigma}\right]\cos\phi d\phi, \tag{1}$$

[a)]Author to whom correspondence should be addressed. Electronic mail: G.palasantzas@phys.rug.nl where the contact factor $P(q,q_{\rm con})$ is given by[5]

$$P(q,q_{\rm con})=\frac{2}{\pi}\int_0^{+\infty}\frac{\sin x}{x}e^{-x^2G(q,q_{\rm con})}dx, \tag{2}$$

$$G(q,q_{\rm con})=\frac{1}{8}\int_{q_{\rm con}}^{q}q^3C(q)dq\int_0^{2\pi}\left|\frac{E(qV\tau\cos\varphi)}{(1-v^2)\sigma}\right|^2 d\phi. \tag{3}$$

The contact factor $P(q,q_{\rm con})$ is the fraction of the original nominal contact area where contact remains when we study the contact area on the length scale $2\pi/q$.[5] In Eqs. (1)–(3), $C(q)$ is the Fourier transform of the auto correlation function $C(r)=\langle h(\mathbf{r})h(0)\rangle$ with $h(\mathbf{r})$ the surface roughness height ($\langle h\rangle=0$). $\langle\cdots\rangle$ is an ensemble average over possible roughness configurations. σ is the applied macroscopic load and $E^*(\omega)$ is the complex conjugate of the Young modulus $E(\omega)$, which is assumed to be given by the rheological-based model[5]

$$E(\omega)=\frac{E_1[(1+\alpha)+(\omega\tau)^2]}{(1+\alpha)^2+(\omega\tau)^2}-j\frac{\alpha\omega\tau E_1}{(1+\alpha)^2+(\omega\tau)^2}, \tag{4}$$

with $E_1=E(\infty)$, and $E(\infty)/E(0)=1+\alpha$ (typically $\alpha=10^3$).[5] $1/\tau$ is the flip rate of molecular segments, which are configuration changes and they are responsible for the viscoelastic properties of the rubber body. Since the flipping is a thermally activated process, we can assume an exponential dependence on temperature in terms of an energy barrier between glassy (high ω) and rubber (low ω) region.[5]

III. RESULTS AND DISCUSSION

As Eq. (1) indicates, in order to calculate the coefficient of friction μ_f the knowledge of the spectrum $C(q)$ is necessary. A wide variety of surfaces/interfaces are well described by a kind of roughness associated with self-affine fractal scaling,[7] for which $C(q)$ scales as a power-law $C(q)\propto q^{-2-2H}$ if $q\xi\gg1$, and $C(q)\propto$const if $q\xi\ll1$.[7] The roughness exponent H is a measure of the degree of surface irregularity,[7] such that small values of H characterize more jagged or irregular surfaces at short length scales ($<\xi$). The self-affine scaling behavior is satisfied by the simple model[8]

$$C(q)=\frac{1}{2\pi}\frac{w^2\xi^2}{(1+aq^2\xi^2)^{1+H}}, \tag{5}$$

with $a=(1/2H)[1-(1+aQ_c^2\xi^2)^{-H}]$ if $0<H<1$ (power-law roughness), and $a=(\frac{1}{2})\ln[1+aQ_c^2\xi^2]$ if $H=0$ (logarithmic roughness).[8] The parameter w is the rms roughness amplitude, and $Q_c=\pi/a_o$ with a_o of the order of atomic dimensions. For other correlation models see also Refs. 9 and 10.

As it is shown in Ref. 5 the factor $P(q,q_{\rm con})$ can be well approximated by the extrapolation formula $P(q,q_{\rm con})=\{1+[\pi G(q,q_{\rm con})]^{3/2}\}^{-1/35}$ which makes calculations of the friction coefficient μ_f simpler. Our calculations were performed for $a_o=0.3$ nm, Poisson modulus $v=0.5$ (ignoring any weak frequency dependence),[5] and relatively weak applied loads σ so that $E_1/\sigma\gg1$. Indeed, as Fig. 1(a) indicates the effect of the ratio E_1/σ becomes more significant when the contact length scale λ becomes large (λ$\gg\xi$), since in this limit the friction coefficient μ_f grows linearly with E_1/σ.[5] In addition, as Fig. 1(b) shows the maximum of μ_f as a function of the sliding velocity V is shifting to higher values with increasing relaxation time τ for both large or small contact length scales. However, the shift of the maximum is relatively smaller for smaller contact length scales [λ$\ll\xi$; Fig. 2(c)].

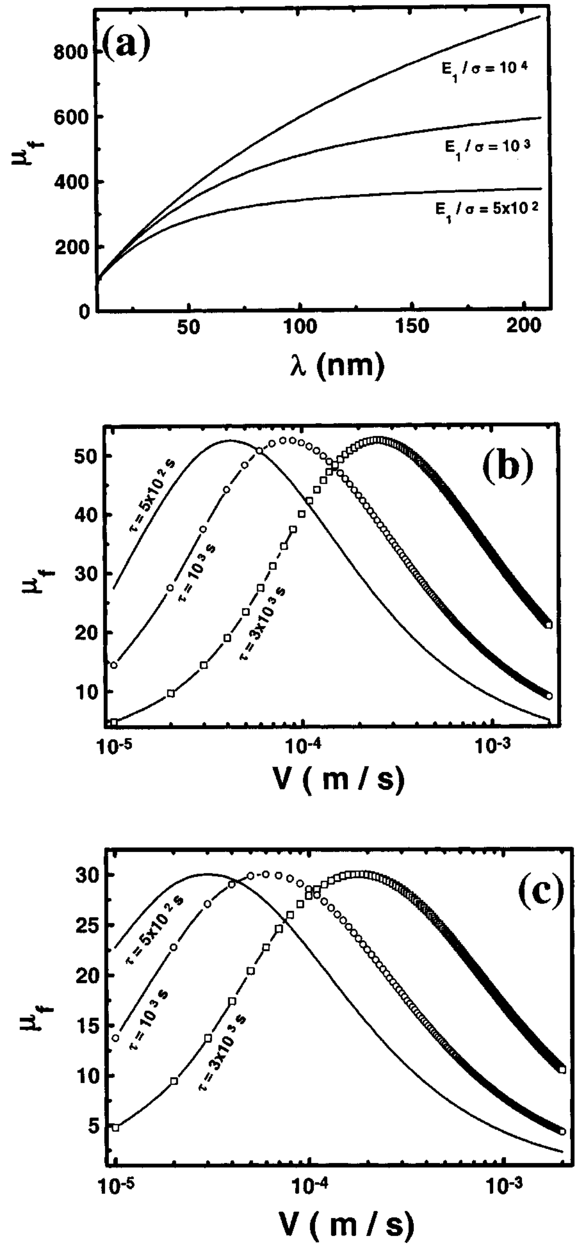

FIG. 1. (a) Friction coefficient μ_f vs contact length scale λ for $V=2\times10^{-4}$ m/s, various ratios E_1/σ, $\tau=10^{-3}$ s, $H=0.8$, $w=5$ nm, and $\xi=100$ nm. (b) Friction coefficient μ_f vs sliding velocity V for λ=30 nm, $E_1/\sigma=1000$, various relaxation times τ, $w=5$ nm, $\xi=100$ nm, and roughness exponents $H=0.8$. (c) The same as in (b) but for λ=10 nm.

Although $C(q)\propto w^2$, as Eq. (5) indicates, the influence of the rms roughness amplitude w on the friction coefficient μ_f under conditions of incomplete contact is more complex than the case of complete contact ($P=1$) where we have $\mu_f\propto w^2$. Therefore, any complex dependence on the substrate surface roughness will arise from all the roughness

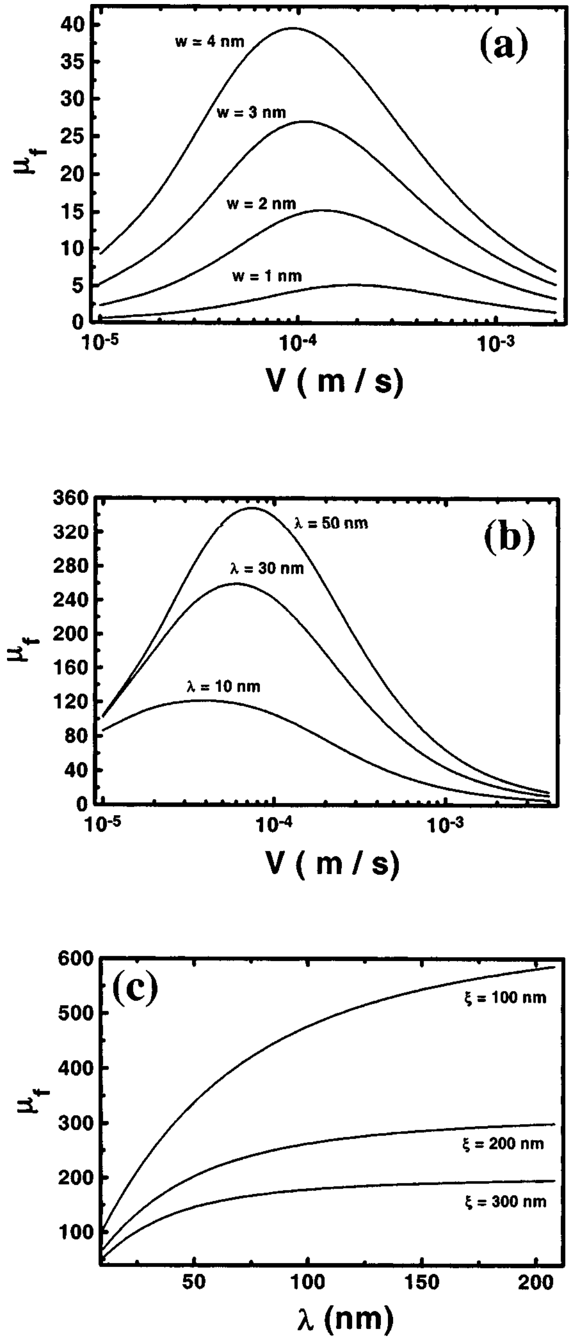

FIG. 2. (a) Friction coefficient μ_f vs sliding velocity V for $\tau = 10^{-3}$ s, $\lambda = 30$ nm, $E_1/\sigma = 1000$, $\xi = 100$ nm, $H = 0.8$, and various roughness amplitudes w. (b) Friction coefficient μ_f vs sliding velocity V for various contact lengths λ, $\tau = 10^{-3}$ s, $E_1/\sigma = 1000$, $w = 5$ nm, $\xi = 100$ nm, and $H = 0.8$. (c) Friction coefficient μ_f vs contact length scale λ for $V = 2 \times 10^{-4}$ m/s, $E_1/\sigma = 1000$, $\tau = 10^{-3}$ s, $H = 0.8$, $w = 5$ nm, and various correlation lengths ξ.

parameters w, H, and ξ. As Fig. 2(a) shows with increasing rms roughness amplitude w, the coefficient of friction μ_f increases, which is intuitively expected. However, the position of the maximum as a function of sliding velocity decreases. The position of the maximum is rather sensitive to changes of the roughness amplitude w as Fig. 2(a) indicates for consecutive values of w. Therefore, the maximum contribution of surface roughness to the friction coefficient μ_f (which occurs around length scales $L = V\tau$) is also dependent on the out-off plane roughness as is expressed by the rms roughness amplitude w.

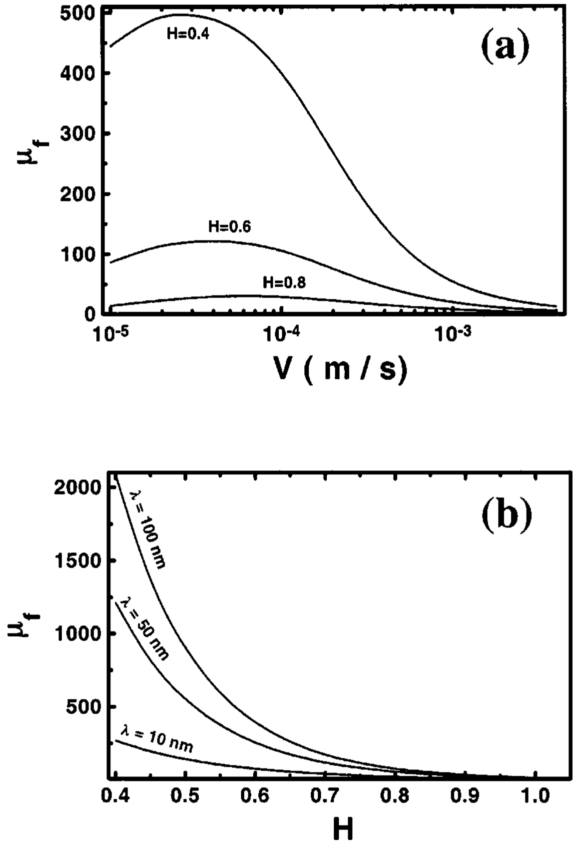

FIG. 3. (a) Friction coefficient μ_f vs sliding velocity V for $\tau = 10^{-3}$ s, $\lambda = 30$ nm, $E_1/\sigma = 1000$, $w = 5$ nm, $\xi = 100$ nm, and various roughness exponents H. (b) Friction coefficient μ_f vs roughness exponent H for $V = 2 \times 10^{-4}$ m/s, $\tau = 10^{-3}$ s, various contact lengths λ, $E_1/\sigma = 1000$, $w = 5$ nm, $\xi = 100$ nm, and $H = 0.8$.

Moreover, with increasing contact length λ [Fig. 2(b)] the friction coefficient increases and the position of the maximum shifts to higher value when λ approaches the lateral correlation length ξ. For $\lambda \gg \xi$, the position of the maximum remains fixed. The effect of the contact length λ clearly becomes more significant around the maximum, where energy dissipation due to internal frictional damping takes place. Alternatively, Fig. 2(c) shows the direct dependence of the friction coefficient on the contact length λ for various correlation lengths ξ.

As a function of the roughness exponent H (Fig. 3), the velocity distribution becomes sharper for smaller exponents H (<0.5; or more jagged rough surfaces at short roughness wavelengths $<\xi$). Clearly the roughness exponent H has a strong influence on the friction coefficient μ_f. The influence of H increases with increasing contact length λ up to values $\lambda \gg \xi$. Notably, the position of the maximum shifts to lower velocities with decreasing roughness exponent H. This is comparable with the behavior as a function of the roughness amplitude w [Fig. 2(a)]. Therefore, with increased surface roughening at short (as expressed by the roughness exponent H) and/or long length scales (as expressed by the roughness parameters w and ξ), the position of the maximum of the friction coefficient μ_f shifts to lower sliding velocities.

Notably, some analytic results can be obtained at low sliding velocities ($V\tau Q_c \ll \alpha$) for the contact factor $P(q,q_{\rm con})$. In this case, Eqs. (3)–(5) yield

$$G(q,q_{\rm con}) \cong \frac{(1+\alpha)^{-2}}{16(1-\nu^2)^2}\left(\frac{E_1}{\sigma}\right)^2 \frac{w^2}{a^2\xi^2}\left[\frac{1}{1-H}\{T_q^{1-H}-T_{\rm con}^{1-H}\} + \frac{1}{H}\{T_q^{-H}-T_{\rm con}^{-H}\}\right], \quad (6)$$

with $T_q=(1+aq^2\xi^2)$ and $T_{\rm con}=(1+aq_{\rm con}^2\xi^2)$. For $G(q,q_{\rm con})\gg 1$ we have the simpler expression $P(q,q_{\rm con}) = 1/\sqrt{\pi G(q,q_{\rm con})}$[5] which further yields

$$P(q,q_{\rm con}) \cong F(a,v)\left(\frac{\sigma}{E_1}\right)\frac{a\xi}{w}\left[\frac{1}{1-H}\{T_q^{1-H}-T_{\rm con}^{1-H}\} + \frac{1}{H}\{T_q^{-H}-T_{\rm con}^{-H}\}\right]^{-1/2}, \quad (7)$$

with $F(a,v)=4(1+\alpha)(1-v^2)/\sqrt{\pi}$. For the limiting cases $H=0$ and 1 one has to employ the identity $\ln(x) = \lim_{c\to o}(1/c)(x^c-1)$ to obtain the proper result. Therefore, we have

$$P(q,q_{\rm con})_{H=0} \cong F(a,v)\left(\frac{\sigma}{E_1}\right)\frac{a\xi}{w}\left[a\xi^2(q^2-q_{\rm con}^2)+\ln\left(\frac{T_{\rm con}}{T_q}\right)\right]^{-1/2}, \quad (8)$$

$$P(q,q_{\rm con})_{H=1} \cong F(a,v)\left(\frac{\sigma}{E_1}\right)\frac{a\xi}{w}\left[\ln\left(\frac{T_q}{T_{\rm con}}\right)+\{T_q^{-1}-T_{\rm con}^{-1}\}\right]^{-1/2}. \quad (9)$$

For higher order terms for the contact factor $P(q,q_{\rm con})$, we have to use in Eq. (2) the expansion $\sin x = \Sigma_{n=0,\infty}(-1)^n x^{2n+1}/(2n+1)!$ (when $G\gg 1$). Thus, we obtain

$$P(q,q_{\rm con}) \cong \frac{1}{\sqrt{\pi}}\sum_{n=0}^{+\infty} 4^{(n+1)}\frac{(1-v^2)^{2n+1}(1+\alpha)^{(2n+1)}}{n!} \times \left(\frac{\sigma}{E_1}\right)^{(2n+1)}\left(\frac{a\xi}{w}\right)^{2n+1}\left[\frac{1}{1-H}\{T_q^{1-H} - T_{\rm con}^{1-H}\}+\frac{1}{H}\{T_q^{-H}-T_{\rm con}^{-H}\}\right]^{-(2n+1)/2}. \quad (10)$$

Finally, for high sliding velocities [i.e., $V>5\times 10^{-4}$ m/s in Fig. 3(a)], the friction coefficient μ_f appears to decrease as a power-law, namely, $\mu_f \propto V^{-\phi}$. The exponent ϕ appears to be a decreasing function of the roughness exponent H. Indeed, as an indicative example, Fig. 4 shows that the exponent ϕ decreases almost in a linear fashion with increasing roughness exponent H. Therefore, surface smoothening at short length scales (higher H) leads to faster decay of the friction coefficient with increasing sliding velocity.

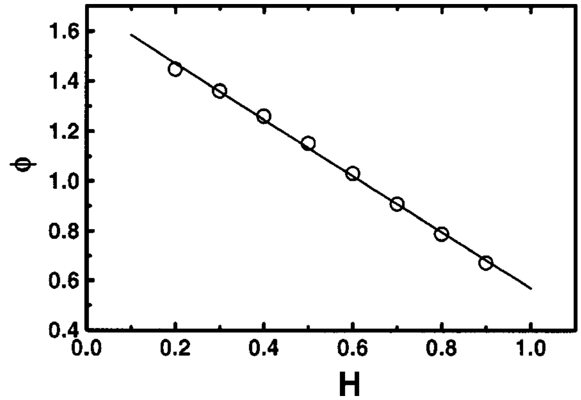

FIG. 4. Exponent ϕ vs roughness exponent H. The estimation of ϕ was obtained for $\tau=10^{-3}$ s, λ=30 nm, $E_1/\sigma=1000$, $w=5$ nm, $\xi=100$ nm, and various roughness exponents H (velocity range $V=1\times 10^{-3}-2\times 10^{-3}$ m/s). The linear fit yields $\phi=1.69-1.13H$.

IV. CONCLUSIONS

In summary, it is shown that with increased surface roughening at short and/or long length scales (decreasing H and/or increasing roughness ratio w/ξ, respectively), the position of the maximum of the friction coefficient μ_f shifts to lower sliding velocities and thus the energy dissipation due to internal frictional damping (which arises from oscillating forces that the surface asperities exert onto the rubber surface). The latter occurs for conditions of incomplete contact or sufficiently small contact length scales ($<\xi$). In all cases, the coefficient of friction increases monotonically with decreasing roughness exponent H and/or increasing roughness ratio w/ξ and attains its maximum value for sufficiently large contact length scales ($\gg\xi$).

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